

A systematic investigation of the link between rational number processing and algebra ability

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### Abstract

Recent research suggests that fraction understanding is predictive of algebra ability, however the relative contributions of various aspects of rational number knowledge are unclear. Furthermore, whether this relationship is notation-dependent or rather relies upon a general understanding of rational numbers (independent of notation) is an open question. In the current study, college students completed a rational number magnitude task, procedural arithmetic tasks in fraction and decimal notation, and an algebra assessment. Using these tasks, we measured three different aspects of rational number ability in both fraction and decimal notation: (1) acuity of underlying magnitude representations, (2) fluency with which symbols are mapped to the underlying magnitudes, and (3) fluency with arithmetic procedures. Analyses reveal that when looking at the measures of magnitude understanding, the relationship between adults' rational number magnitude performance and algebra ability is dependent upon notation. However, once performance on arithmetic measures is included in the relationship, individual measures of magnitude understanding are no longer unique predictors of algebra performance. Furthermore, when including all measures simultaneously, results revealed that arithmetic fluency in both fraction and decimal notation each uniquely predicted algebra ability. Findings are the first to demonstrate a relationship between rational number understanding and algebra ability in adults while providing a clearer picture of the nature of this relationship.

**Keywords:** *Decimals; Fractions; Numerical Magnitude Knowledge; Procedural Knowledge;*

*Algebra*

A Systematic Investigation of the Link Between  
Rational Number Processing and Algebra Ability

Understanding rational numbers is a critical building block for advanced scientific and mathematical thinking. Even when controlling for other math abilities, fraction and decimal knowledge is a unique predictor of arithmetic proficiency and more general math achievement (Bailey, Hoard, Nugent, & Geary, 2012; Schneider, Grabner, & Paetch, 2009; Siegler, Thompson, & Schneider, 2011; Siegler & Pyke, 2013). Furthermore, rational number understanding has been linked specifically to algebra readiness (Booth & Newton, 2012) and algebra ability in high school (DeWolf, Bassok, & Holyoak, 2015; Siegler et al., 2012). However, the nature of this relationship is not fully understood. Clearly, algebra ability relies upon a number of important skills – an ability to manipulate symbols, a firm understanding of the number system, and a strong conceptual understanding of how arithmetic works (e.g., Carraher, Schliemann, & Brizuela, 2000; Fuchs et al., 2008; Linchevski, 1995) – all skills that are strengthened through working with rational numbers. However, whether the relationship between rational number understanding and algebra ability rests primarily upon a better conceptual understanding of the procedures involved in higher-order mathematics or instead upon a deeper understanding of the continuous nature of our numerical magnitude system is a relatively open question. In line with suggestions from the National Mathematics Advisory Panel (2008) and the Common Core State Standards (National Governors Association Center for Best Practices, 2010) which outline the importance of understanding the relationship between rational number magnitude understanding, arithmetic ability, and algebra ability, in the current study we investigate the relative importance of rational number magnitude processing (both precision in the underlying representation and fluency in understanding how symbols are mapped to

numerical magnitudes) and of rational number procedural competence in predicting algebra ability in young college students.

### **Rational Number Magnitude Understanding**

One possibility is that algebraic achievement may rely upon a precise understanding of the continuous numerical magnitudes that fall between integer values (i.e., rational number magnitudes) and/or fluency with mapping between symbolic notation (decimal and/or fraction notation) and the analog mental magnitudes they represent. These abilities, which are generally assessed using number comparison tasks in which participants are asked to rapidly judge which of two numerical symbols is larger and/or with number-line tasks in which participants are asked to place a number along a line with two numerical endpoints, have been shown to be strongly predictive of mathematical achievement (e.g., Bugden & Ansari, 2011; Holloway & Ansari, 2009). For example, 1<sup>st</sup> and 2<sup>nd</sup> grade children's symbolic whole-number comparison performance has been shown to predict their more general math achievement (Bugden & Ansari, 2011). However, the link between performance on magnitude tasks and math achievement is not constrained to whole-number magnitudes; more recently, rational number magnitude judgments have also been shown to predict general math ability. For example, when participants are asked to indicate where a fraction or decimal falls on a number line (with endpoints of 0 and 1) and/or to rapidly compare the relative magnitude of two fractions or decimals, performance on these magnitude tasks is often related to more general math achievement (Booth & Siegler, 2008; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider, Grabner, & Paetch, 2009; Siegler, Thomson, & Schneider, 2011; Siegler & Pyke, 2013) as well as algebra readiness (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014).

But what exactly is it about performance on these tasks that tap into later mathematical

understanding? Importantly, both number line and comparison tasks arguably assess two distinct aspects of magnitude processing: (1) the precision in the underlying representation of rational numbers (i.e., numerical acuity) and (2) the automaticity or fluency of the mapping between numeric symbols (i.e., fractions, decimals) and the numerical magnitudes that they represent. Previous research has linked both of these aspects of magnitude processing to general math ability for whole numbers (Castronovo & Gobet, 2012; De Smedt, Verschaffel, & Ghesquiere, 2008; Geary, 2011; Mundy & Gilmore, 2009). In the current study, we explore the potential contribution of each of these aspects of *rational number* magnitude processing to algebra achievement, in particular.

*Magnitude Acuity:* Number magnitude acuity, or the precision in the underlying representation of number magnitude, has repeatedly been found to correlate with formal math measures. The acuity of underlying magnitude representations is typically measured via number comparison tasks in which individuals are asked to rapidly judge which of two numbers is largest. Numerical comparisons of whole numbers and non-whole numbers obey Weber's law, such that the discriminability of two values is dependent upon their ratio (e.g., Hurst & Cordes, 2016; Moyer & Landauer, 1967, 1973); that is, the closer two values are, the longer it takes to identify the larger value. This ratio-dependence of numerical discriminations gives rise to "ratio effects" in the behavioral data, assessed as the slope (i.e.,  $\beta$  estimate) of the line relating response time to the ratio of the two values being compared<sup>1</sup>. Prominent models of numerical representation posit that individual ratio effects serve as a proxy for the precision in the

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<sup>1</sup> The ratio of values was computed as the larger value / smaller value, with higher ratios involving greater relative differences between the values presented (resulting in faster comparisons). When measured in this way, ratio effects (i.e., slopes) are negative values, however throughout the current manuscript we will refer to "strong" ratio effects as being those values that are highly negative (i.e., far away from zero in the expected direction).

underlying representation of numerical magnitudes, with strong ratio effects reflecting less precision in the underlying representation (e.g., Holloway & Ansari, 2009; Moyer & Landauer, 1967, 1973).

In line with the hypothesis that representational precision is important for math achievement, many studies have shown that strong individual whole number ratio effects negatively correlate with math achievement (e.g., Castronovo & Gobel, 2012; De Smedt, Verschaffel, & Ghesquiere, 2008; Geary, 2011; Halberda, Mazocco, & Feigenson, 2008; Holloway & Ansari, 2009; Mundy & Gilmore, 2009) and positively correlate with math anxiety (Maloney, Ansari & Fugelsang, 2011). Furthermore, having a precise representation of numerical magnitudes is posited to be important for accurately approximating answers to basic arithmetic (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Gilmore, McCarthy, & Spelke, 2007). However, whether individual ratio effects obtained from rational number magnitude tasks are similarly predictive of algebra performance is an open question.

Recent work also suggests that, beyond the point estimate of an individual's ratio effect (i.e., slope,  $\beta$ ), variability in the measurement of each individual's ratio effect (i.e., the standard error (SE) of each individual's slope estimate) may also be important for predicting math ability (Lyons, Nuerk, & Ansari, 2015). This measure, a distinct but related measure of precision in the individual's underlying representation of number, has been shown to be a more consistent predictor of arithmetic ability in children than the point estimate of their ratio effect in the case of whole numbers. Thus, given that both ratio effects and the variability in ratio effect measurements of whole numbers have been shown to predict general math achievement, and other work has shown a link between rational number understanding and algebra ability, it may be that a precise representation of rational number magnitudes is similarly important for algebra

performance, reflecting a better understanding of the continuum of rational numbers and thus a better sense of the range of values that unknown variables may represent.

Furthermore, adults' understanding of rational number magnitudes may not be equivalent across distinct notations. In particular, evidence suggests that it is substantially easier to access magnitude information from decimals than from fractions (DeWolf, Grounds, Bassock, & Holyoak, 2014; Hurst & Cordes, 2016), and thus magnitude understanding in these two notations may not be equally predictive of algebra ability. For example, although fraction notation may be a more complicated symbol (given its componential nature that is very different from the typical place-value system), understanding magnitude information using decimals may provide adults with a more direct understanding of rational number magnitudes. In the current study, we explore this possibility by determining whether rational number magnitude acuity (as assessed via ratio effects (slope,  $\beta$ ) and variability in ratio effects (standard error, SE) in numerical comparison task data) in decimal notation and in fraction notation, together or separately, predicts algebra ability.

*Symbolic Magnitude Fluency:* Regardless of the level of precision in the underlying magnitude representation, in order to work with rational number notation, a fluent mapping between the symbolic numerals (i.e., fractions, decimals) and the underlying numerical magnitudes that they represent must be acquired. That is, working with rational numbers requires a sense of what numerical magnitude is denoted by, for example "5/8". As such, symbolic magnitude fluency has been assessed as how well an individual is able to map between a symbol and the symbol's representation (for example, as the average speed in a numerical comparison task). Importantly, whole number symbolic mapping has been found to predict math achievement (Castronovo & Gobel, 2012) even when controlling for other aspects of magnitude

understanding (Mundy & Gilmore, 2009). It is quite possible a similar relationship may hold between rational number symbolic magnitude fluency and algebraic ability, particularly due to the complex notation used for rational numbers (involving both Arabic numerals as well as non-numerical symbols (i.e., the period in decimals and the vinculum or dividing line in fractions)). Thus, an automatic mapping from the complex fraction and decimal notation to the underlying representation of magnitude requires a flexible use of symbols and symbol formation, a skill also required for symbolic manipulation in algebra (Swafford & Langrall, 2000). As such, rational number symbolic magnitude fluency may be an important predictor of algebraic skill, as they both reflect an ability to think about the meanings behind abstract symbols. In the current study, we explore this relationship.

### **Rational Number Arithmetic Procedural Fluency**

In addition to magnitude understanding of rational numbers, it may be that the ability to execute procedures with fractions and decimals is predictive of algebra ability. Some evidence does suggest that performance on fraction arithmetic assessments is related to general math ability (Bailey, Hoard, Nugent, & Geary, 2012), but it is unclear whether this relationship holds when predicting more advanced mathematical thinking, such as algebra. However, there is reason to think that performance on rational number arithmetic assessments may be particularly important for algebra. Algebra requires an ability to quickly and flexibly manipulate symbols in order to manipulate equations and perform calculations. As such, algebra scores have been found to correlate with non-numerical symbolic abilities, including those necessary for understanding the syntax of language (MacGregor & Price, 1999), suggesting an ability to follow abstract rules and/or manipulate even non-numerical symbols is an important contributor to algebra ability. Furthermore, beyond pure symbol manipulation, successfully performing arithmetic with

fractions and decimals requires substantial conceptual understanding of the way arithmetic works, a skill that is also important for solving complex algebra. Thus, in the current study we also explore the contribution of procedural understanding, for decimal arithmetic and fraction arithmetic, in predicting algebraic performance.

### **Educational Experience**

Notably, previous reports of a link between rational number understanding and algebra achievement (DeWolf, Bassok, & Holyoak, 2015; Siegler et al., 2012) have been limited to exclusively exploring this relationship in children, who are still in the process of acquiring these mathematical concepts. Given that mathematical learning is a slow, extended process throughout adolescence, it is important to determine whether, once rational number and basic algebra course instruction is complete by the time students reach college, the relationship between rational number understanding and algebra still holds. On the one hand, by adulthood, individuals may have developed strategies that help them to circumvent the limitations brought on by poor fraction understanding, such as, for example, relying more heavily upon decimal notation when performing advanced mathematics. If so, these alternative strategies may mute the strength of the relationship between fraction understanding and fluency with basic algebra. On the other hand, given that difficulties in rational number processing (in both fraction and decimal notation) persist into college-aged students (e.g. DeWolf et al., 2014; Ni & Zhou, 2005), there is reason to believe that the relationship between fraction knowledge and algebra abilities established in childhood may be indicative of an overall competence with advanced mathematical concepts and as such, the relationship should hold into adulthood with basic algebra fluency. Thus, investigating the mechanisms involved in the relationship between rational number understanding and algebra in a group of young college-educated adults may provide important

insights into the strategies and performance patterns shown in experienced learners, who have completed formal schooling in these topics.

### **The Current Study**

In the current study we aimed to further specify the relationship between rational number understanding and basic algebra achievement using three specific measures of rational number understanding in a group of educated adults. Using a magnitude comparison task we assessed (1) the fluency of the mapping between fraction and decimal symbolic notation and the magnitudes they represent (symbolic magnitude fluency) and (2) the precision of the underlying magnitude representation (numerical acuity, both via ratio effect slopes ( $\beta$ ) and variability in the ratio effect estimates (SE)). In addition, adults completed measures of fraction and decimal arithmetic to assess rational number arithmetic fluency in these distinct notations. Since previous work has rarely investigated both decimal and fraction notation within the same study, we included measures of both notations in order to determine the relative contributions of rational number knowledge in each symbolic notation. This research addresses four open questions: (1) Does the relationship between rational number ability and algebra hold in young college-educated adults even after learning the required math content? (2) Does the relationship between magnitude understanding and algebra fluency depend on the type of magnitude understanding (symbolic magnitude fluency versus precision) and/or the notation of the symbolic magnitude representation (fractions versus decimals)? (3) Does the relationship between rational number arithmetic and algebra depend on the notation used? And (4) when both magnitude and arithmetic measures are included in the relationship to predict algebra fluency, what is the relative contribution of each measure?

### **Method**

## Participants

Fifty-one college students (M=19.4 years, Range: 17 to 24 years, 37 females) participated for \$10 or course credit and were included in all analyses. Data from an additional twelve participants were excluded for experimenter or computer error (3), for not meeting the required criteria on the number comparison task (6), or for not meeting the required criteria on the assessments (3). See “Data Analysis” for details of exclusion criteria.

## Stimuli and Apparatus

### *Algebra Assessment*

Twelve basic algebra questions were adapted from the released questions for the Trends in International Mathematics and Science Study, Grade 8 level assessment (TIMSS, 2003, 2011; see Appendix A for a complete list of questions) by removing the multiple-choice options, making them open-ended response questions. Questions included simplifying or solving expressions (e.g., simplify:  $4x-x+7y-2y$ ; solve  $2a+3(2-b)$  given  $a=3$ ,  $b=-1$ ) and understanding relations (e.g., given a table of  $x$  and  $y$  values, write the equation relating  $y$  to  $x$ ). The entire assessment had a Cronbach’s Alpha of 0.77 (based on time to complete each question). Questions were presented in a random order. Importantly, none of the algebra questions included any non-integer rational number knowledge (i.e., did not involve fractions or decimals).

### *Number Comparison Task*

The number comparison task was presented on a SensoMotoric Instruments (SMI; Boston, MA) mobile Eye Tracker, with a  $1400.08\text{cm}^2$  (22-inch) screen (1024 x 768 px). The task involved three distinct blocks of trials, in which participants were required to compare the relative magnitude of two fractions (FvF block), two decimals (DvD block), or two whole numbers (NvN block). In order to measure an individual’s precision with which they represent

rational number magnitudes we manipulated the ratio between the two to-be-compared numbers, allowing us to measure ratio effects (e.g., DeWolf, Grounds, Bassock, & Holyoak, 2014; Moyer & Landauer, 1967; Schneider & Siegler, 2010), which are reported elsewhere (XXX, 2016).

Thus, each block of notation-specific comparisons included four unique comparisons from each of four approximate ratio bins: 1.125 bin (range 1.11 to 1.17), 1.25 bin (range 1.24 to 1.27), 1.5 bin (range 1.35 to 1.67), and 2.5 bin (range 2.2 to 2.92). Each unique comparison was shown twice (once with the largest value on the right and once with the largest value on the left), resulting in 32 trials in each of the FvF, DvD, and NvN blocks (4 ratios x 4 unique comparisons x 2 (shown twice)), making a total of 96 trials (32 x 3 blocks) across all three blocks of trials.

All numerical stimuli were created in Arial regular font size 72pt (approximately 2cm high). The fixation cross was in 32pt font (1cm<sup>2</sup>). The fraction stimuli were designed in order to prevent the use of overt whole number strategies on the fraction components (i.e., comparing only numerators or only denominators; Schneider & Siegler, 2010). In particular, no fraction pair contained the same natural number in more than one component, meaning each fraction comparison contained four distinct natural numbers ranging from 1 to 15 (e.g., 3/4 versus 4/7 would not occur). Furthermore, fraction pairs were congruent with the numerators (i.e., the larger fraction was consistent with the larger numerator) on 14 of the 32 trials and incongruent on the other 18 trials, making exclusively numerator-based comparison strategies unreliable. For the decimal stimuli, all numbers contained a whole number before the decimal point and two digits after the decimal point (e.g., 0.20; 1.56) in order to prevent responding based on decimal length. Notably however, because decimal length was not manipulated, this may have allowed for the use of other, non-magnitude based strategies, such as whole number based processing (i.e.,

treating a decimal value as a whole number, such as treating the comparison 0.51 vs. 0.38 as 51 vs. 38). Decimal values ranged from 0.20 to 22.50 and fraction values ranged from  $1/5$  to  $15/2$ .

### *Fraction and Decimal Arithmetic*

The assessments were presented in two separate blocks, with each arithmetic assessment containing 8 items (two items each of addition, subtraction, multiplication, and division). On the decimal assessment, one problem of each arithmetic type (4 total) involved two decimals to the hundredths digits (e.g.,  $1.27 + 0.89$ ) and the other problem involved one decimal to the hundredths digit and the other to either the tenth or thousandths digit (e.g.,  $0.5 + 0.13$ ;  $1.74 - 1.321$ ; 4 problems total of mixed length). On the fraction assessment, none of the problems contained fractions with the same denominator. Answers to all problems resulted in positive values. Questions were presented in a random order.

### **Procedure**

Participants were seated alone in a quiet room. The experimenter entered the room to explain the instructions and answer questions at the beginning of each task and then left during the tasks. Participants completed the tasks in the following order: (1) Algebra assessment, (2) Number Comparison task, and (3) Fraction and Decimal assessments (order counterbalanced). For all tasks, participants were encouraged to perform as quickly and accurately as possible.

For the three math assessments, participants were seated in front of a Macintosh laptop computer and given a pen and a blank workbook. Questions were presented one at a time on the computer screen, but participants were provided with a workbook and pen to work out the solution and provide the answer. In order to advance to the next question, the participant clicked a button on the computer screen. Participants were told they had as much time as they needed, but that they were being timed and to work as quickly as they could.

The comparison task was performed on a different computer that also recorded eye-movements and began with an eye-tracking calibration procedure (eye-tracking data presented elsewhere; XXX, 2016). Participants were presented with a set of three blocks of notation-specific trials in which they compared two Fractions (FvF; e.g.,  $1/2$  vs.  $3/4$ ), two Decimals (DvD; e.g., 0.50 vs. 0.75), or two Whole Numbers (NvN; e.g., 5 vs. 2) and were asked to select which of the two numbers was larger as quickly and accurately as possible. No feedback was provided. Participants were also presented another set of three blocks of 32 trials each involving across-notation comparisons (i.e., Decimal vs. Fraction; Decimal vs. Whole Number; Whole Number vs. Fraction). Data from those additional blocks are discussed elsewhere (XXX, 2016). These additional blocks were either performed before or after the set of within-notation blocks (order counterbalanced across participants)<sup>2</sup>.

Using their right hand, participants selected one of the two neighboring keys on the keyboard to respond to which number was larger. Participants advanced to the next trial by using their left hand to push the F4 key on the keyboard. Participants were instructed to keep both hands on the keyboard throughout the session. Numbers remained on the screen until the participant advanced to the next trial. Between each trial, a fixation-cross appeared in the middle of the screen to direct attention back to the middle. Participants performed two non-numerical practice trials in which they rapidly selected the side containing an image of a circle. In addition, each block of test trials began with two condition specific practice trials, with feedback from the computer.

## Data Analyses

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<sup>2</sup> There was no difference in performance on the FvF, DvD, or NvN blocks when they were presented in the first half versus in the second half of the six blocks ( $p$ 's > 0.4), suggesting that these additional blocks did not impact performance.

*Algebra and Procedural Assessments:* Accuracy on all assessments was fairly high with relatively low variability (Fraction Arithmetic (score out of 8)  $M=7.04$ ,  $SD=1.3$ ; Decimal Arithmetic (score out of 8)  $M=6.08$ ,  $SD=1.3$ ; Algebra (score out of 12)  $M=10.44$ ,  $SD=1.39$ ). In particular, over 50% of participants obtained a perfect score on the Fraction Arithmetic. Thus, for each of the assessments the total time spent on the assessment (Completion Time; CT) was used as a measure of fluency and the dependent variable. In order for this measure of fluency to be a valid proxy for ability, participants who provided incorrect responses on more than half of the problems in each assessment were excluded from the analysis (resulting in three excluded participants)<sup>3</sup>.

*Number Comparison Task Measures:* We obtained three different measures from the magnitude comparison task: two measures of magnitude acuity (ratio effect point estimate, (slope, or  $\beta$ ), and variability of this estimate (SE) as in Lyons et al., 2015) and one measure of symbolic magnitude fluency (an adjusted average reaction time). For all calculations, only reaction times (RT) from correct responses and those within 3 standard deviations of the individual's mean for that block were included.

To measure **magnitude acuity**, a regression analysis was performed for each individual participant, treating each trial as an observation. RT was regressed onto ratio bin for each notation separately (decimal (DvD trials) and fraction (FvF trials)) to get a point estimate of the individual's ratio effect ( $\beta$ ) for each notation as well as a measure of variability of this point estimate (SE: the standard error of  $\beta$ ). In order to be included in the slope measures, individuals needed to have at least 2 useable RT measures and to have scored above chance for at least 3 of the 4 ratio bins. In other words, if data were excluded for more than one ratio bin, that

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<sup>3</sup> Moreover, when accuracy and completion time are combined to form an IES score (Townsend & Ashby, 1983) the pattern of results obtained is very similar.

participant's ratio effect and SE of ratio effect was not calculated. This resulted in the exclusion of data from 5 individuals. One additional participant was excluded for having excessively long RTs on the majority of trials (>50% of responses on Fraction trials were longer than 10 seconds).

**Symbolic magnitude fluency** was measured as the speed with which individuals processed rational number values. In order to isolate the variation in performance specific to fraction and decimal symbols (and not from magnitude fluency more generally), we controlled for general magnitude processing by subtracting each individual's speed of processing (average RT) on the whole number comparison task from their speed of processing (average RTs) on the decimal and fraction comparison tasks. To compute this measure, average RT in responding (across the four ratio bins) for FvF, DvD, and NvN trials were calculated separately. Then, in order to isolate rational number magnitude processing over and above basic processing speed, we calculated an adjusted RT ( $RT_{adj}$ ) measure for DvD and FvF trials by subtracting the individual's average RT on NvN trials (a proxy for basic magnitude comparison and processing speed) from the DvD and FvF trials ( $FvF RT_{adj} = (FvF RT) - (NvN RT)$ ;  $DvD RT_{adj} = (DvD RT) - (NvN RT)$ ). Thus any variability in the resulting  $RT_{adj}$  measures can be accounted for by difficulty in processing numerical magnitudes presented in decimal or fraction notation, and does not represent differences in fine motor control and/or basic processing speed.

*Outliers:* At the group level, any values that were identified as being more than 3 standard deviations away from the group mean were replaced with the next value that was within the 3 standard deviation criterion. This occurred for five data points (two Decimal Procedural CTs and one data point in each of the decimal magnitude measures ( $\beta$ , SE,  $RT_{adj}$ )).

## Results

Both FvF and DvD comparisons resulted in significant ratio effects (significantly negative slopes; details reported in XXX, 2016) suggesting that on average, adults accessed the approximate magnitudes represented by the symbols during the magnitude comparison task and as anticipated, performance was more difficult for narrower ratios.

First, we looked at the pattern of bivariate correlations (presented in Table 1, along with descriptive statistics) for each of the three magnitude measures ( $\beta$ , SE, and  $RT_{adj}$ ) for Fractions and Decimals separately. For Fractions, all three measures were significantly correlated. In particular, people with *stronger* ratio effects had *higher* variability in their ratio effects (SE) and *lower* symbolic magnitude fluency (i.e., longer  $RT_{adj}$ ). Thus, stronger ratio effects for Fractions seem to correspond with slower and less consistent responding, consistent with the notion that stronger ratio effects are indicative of poorer understanding.

Decimals, on the other hand, showed a different pattern. Consistent with fractions, lower symbolic magnitude fluency (i.e., longer  $RT_{adj}$ ) was associated with more variability in responding (higher SE). However, ratio effects ( $\beta$ ) for decimals were not significantly correlated with any other magnitude measures (symbolic magnitude fluency ( $RT_{adj}$ ) or variability (SE)).

Correlations with Algebra fluency revealed that all rational number measures except FvF  $\beta$  were significantly correlated with Algebra fluency. In particular, adults with lower arithmetic fluency (i.e., took longer to complete both arithmetic assessments), lower symbolic magnitude fluency (higher response times on both comparison tasks;  $RT_{adj}$ ), higher variability in their ratio effects (higher SE) for both decimal and fraction notation, and lower decimal (but not fraction) ratio effects (i.e., slope ( $\beta$ ) closer to zero) took longer to complete the Algebra assessment. Fraction ratio effects were not significantly related to Algebra fluency.

Notably, the measures of variability in ratio effects (SE) for both fraction and decimal notation were highly correlated with symbolic magnitude fluency ( $RT_{adj}$ ; Fractions:  $r=0.9$ ; Decimals:  $r=0.6$ ), consistent with previous research (Lyons et al., 2015). Because these high correlations (particularly for fractions) would have led to issues with multicollinearity in the regression and because previous research has shown that average RT is a better predictor of math performance than SE (Lyons et al., 2015), we included  $RT_{adj}$ , but not the SE measures, in the combined regression analyses.

Thus, in order to investigate the relative contribution of various measures of fraction understanding to algebra ability, we used a regression analysis using the magnitude measures (Model 1) and the arithmetic measures (Model 2) as predictors of Algebra fluency. Table 2 provides all of the statistics for both individual models as well as the combined model.

First, to test the additional contribution of the two arithmetic measures over and above the magnitude measures, we entered the magnitude measures (DvD  $RT_{adj}$ , FvF  $RT_{adj}$ , DvD  $\beta$ , FvF  $\beta$ ) in the first step and entered the arithmetic measures (Decimal and Fraction arithmetic fluency) in the second step. Results indicated that the model including only magnitude measures was statistically significant, with both decimal measures (DvD  $\beta$  and DvD  $RT_{adj}$ ) providing unique statistical significance ( $p < 0.05$ ; and marginally FvF  $RT_{adj}$ ,  $p = 0.066$ ). Importantly, however, the addition of the arithmetic measures was also significant,  $\Delta R^2 = 0.307$ ,  $F(2,44) = 16.8$ ,  $p < 0.001$ .

Second, to investigate the additional contribution of the set of magnitude measures, over and above the arithmetic measures, we conducted a second regression in which we entered the arithmetic predictors in the first step and added the set of magnitude measures in the second step. Not surprisingly, the model with only the arithmetic measures was significant, with both decimal

and fraction arithmetic each explaining statistically significant unique variance ( $p$ 's $<0.001$ ). The addition of the set of magnitude measures resulted in statistically significant change in the amount of variance explained by the model overall,  $\Delta R^2=0.103$ ,  $F(4,44)=2.8$ ,  $p=0.037$ .

Finally, beyond the overall impact of the set of magnitude measures and the set of arithmetic measures, we investigated the impact of each individual measure on algebra performance by looking at the unique contribution of each measure in the overall, combined model. None of the individual magnitude measures explained significant unique variance, when controlling for all the others, although both Decimal measures showed small, marginally significant effects ( $p<0.1$ ). On the other hand, both fraction and decimal arithmetic fluency explained significant unique variance ( $p$ 's $<0.05$ ) above and beyond the magnitude measures as well as each other.

### **Discussion**

The relationship between rational number understanding and algebra ability is complex. Our aim was to further characterize this relationship by exploring the relative contribution of various aspects of rational number understanding. In doing so, our results demonstrate novel predictors of algebra ability that are dependent upon the format of the symbolic notation used to measure them.

First, our results reveal that the relationship between rational number arithmetic and algebra previously reported in children (e.g., Siegler et al., 2012) also holds in educated adults (college students) who have completed their schooling in rational number and basic algebra concepts. Rational number understanding thus remains an important predictor of algebra ability long after these skills have been acquired, suggesting that this relationship is not dependent upon recent instruction of these concepts.

More importantly, our findings further clarify the relationship between rational number understanding and algebra performance by suggesting that the relationship does depend on the type of knowledge being measured and the notation being used. When looking at magnitude predictors, we found that overall rational number magnitude understanding (as assessed by  $RT_{adj}$  and  $\beta$  for both decimals and fractions) significantly predicted algebra fluency – however, only decimal (not fraction) magnitude acuity and symbolic magnitude fluency were uniquely predictive. In particular, higher algebra fluency was associated with a higher fluency with symbolic decimal magnitudes and with stronger decimal ratio effects. That is, adults who were quicker at the algebra assessment were faster at processing decimal notation and were more impacted by the ratio of the magnitudes involved in the comparison task. The direction of this relationship with ratio effects is in contrast to typical results with whole numbers, such that weaker whole number ratio effects (typically attributed to more precise representations) are associated with better math ability (Holloway & Ansari, 2009) and lower math anxiety (Maloney, Ansari & Fugelsang, 2011). Although this finding may be counterintuitive, it opens up the possibility that the interpretation of ratio effects for whole number magnitudes may not be equally applicable for rational number comparisons. In particular, the existence of ratio effects for decimals comparisons has only recently been explored, with several studies showing that other, non-magnitude based strategies may interfere with magnitude dependent responding (Bonato et al., 2007; Desmet, Gregoire, & Mussolin, 2010; Kallai & Tzelgov, 2014; Varma & Karl, 2013). For example, whereas the goal of a magnitude comparison task is for participants to compare the numerical magnitudes associated with the given numbers, when presented with two decimals, it is possible for participants to engage in strategies such as comparing the length of decimal values or comparing the values of individual digits (i.e., comparing tenths, then

hundredths, etc.), which may obscure any possible ratio effects in magnitude judgments. In our sample, we found approximately 30% of participants had a weak or non-existent decimal ratio effect (i.e., a positive or near-zero  $\beta$  value). It is possible that these participants engaged alternative strategies that did not rely exclusively upon numerical magnitude. On the other hand, those individuals with stronger ratio effects may have consistently used magnitude-based strategies. If so, then in contrast to ratio effects observed in whole-number comparisons, which are thought to be a measure of precision of the underlying representation, *decimal* ratio-effects may instead indicate whether or not the individual used magnitude-based processing. As such, it may be that fraction and decimal ratio effects follow a u-shaped curve with positive or near-zero ratio effects indicative of non-magnitude based responding (associated with poor math performance) and highly negative ratio effects indicative of imprecision in the representation of rational number magnitudes (again, associated with poor performance), with some “ideal” level of ratio dependent responding falling in-between. This theory may be most appropriately explored through future developmental investigations. Most importantly, this pattern of findings further highlights the need for investigating the representation of magnitudes presented across distinct notations simultaneously in order to shed light on how symbolic notation may convey magnitude information in different ways.

In the current study, we used a numerical magnitude comparison task in order to provide a measure of rational number magnitude precision and fluency. Although comparison tasks are widely used in the literature to assess whole number magnitude understanding, many researchers have also employed number line tasks, in which individuals are asked to place a number along a line with two numerical endpoints (e.g., Iuculano & Butterworth, 2011). While substantial research and debate has focused on what aspect(s) of numerical magnitude understanding

number line estimation tasks are measuring (for example, the format of the underlying mental representation versus proportion based responding; e.g., Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011; Hurst, Monahan, Heller, & Cordes, 2014; Opfer, Siegler, & Young, 2011), similar to the results from the current study using a comparison task, performance on number line tasks have also been shown to relate to more advanced math ability, including algebra (Booth & Newton, 2012; Siegler, Thompson, & Schneider, 2011). Thus, future research should further investigate performance on a variety of magnitude-based tasks, using different notations, and presenting them in different formats (e.g., different length decimals, same length decimals, different denominators, same denominators, etc.) to provide converging evidence to further our understanding of how people approach rational number values in each of these contexts and how performance may be associated with better or worse understanding.

Interestingly, although there was some evidence that measures of rational number magnitude understanding were predictive of algebra ability, once arithmetic fluency measures were controlled for, magnitude measures provided only a small amount of additional variance (about 10%) in predicting algebra ability and none of the individual magnitude measures uniquely predicted algebra ability. Thus, it may be that, once arithmetic ability was controlled for, our magnitude measures served as a proxy for a general understanding of non-integer, rational number values, in which the format of the notation was irrelevant.

On the other hand, the arithmetic measures accounted for 31% of the variance in algebra fluency, even after controlling for the magnitude measures. One explanation for why arithmetic fluency accounted for such a high amount of variance in algebra ability may be that the arithmetic fluency measures also encompassed some aspect of magnitude understanding. That is, rational number arithmetic fluency requires some of the same abilities to process rational number

notation and potentially to approximate magnitudes that a comparison task may entail. Thus our measure of arithmetic fluency may have also partially accounted for measures of magnitude understanding. In addition, unlike our magnitude measures in which we adjusted for general speed of processing of numerical information, we did not have a measure of whole-number arithmetic fluency to subtract from the rational number arithmetic measures, thus confounding our rational number arithmetic measures with general arithmetic processing (i.e., the speed with which individuals perform arithmetic, regardless of the format of the numbers involved). Future research should also include general measures of arithmetic and magnitude processing in order to account for the potential overlap in these tasks. In general, however, these findings do suggest that the fluency and flexibility with complex notations involving fractions and decimals as magnitudes and in performing arithmetic may be key variables involved in the relationship between rational number understanding and algebra ability.

Importantly, the algebra assessment did not require any computations involving fraction or decimal values, suggesting that the obtained relationship between algebra and rational number arithmetic was not solely driven by the individual's ability to arithmetically manipulate fractions and decimals specifically. Moreover, since both fraction and decimal arithmetic fluency measures were significant predictors while controlling for the other, the underlying relationship was not entirely due to factors that are shared between both fraction and decimal arithmetic. Thus, the relationship is not solely due to arithmetic fluency and the memorization of procedures in general, since this would be a shared skill between fraction and decimal arithmetic (although these findings do not rule out that this skill is involved, just that it is likely not the only relevant factor). Instead, factors unique to fraction arithmetic fluency and unique to decimal arithmetic fluency are likely each related to algebra fluency. For example, fraction arithmetic requires

knowledge about denominators and ratios and decimal arithmetic requires knowledge about place-value, both factors that may play a role in algebra learning.

Although algebra performance appears to uniquely rely upon aspects of fraction arithmetic and of decimal arithmetic, more research is needed to clarify what aspects of arithmetic processing is crucial for algebraic understanding. In particular, arithmetic relies upon both conceptual knowledge of arithmetic processes (e.g., understanding why common denominators are required for addition, but not multiplication) as well as a procedural understanding of the actions to be carried out (e.g., knowing the procedure for finding common denominators). Furthermore, conceptual and procedural learning within the domain of rational numbers tend to be very intertwined; for example, an improvement in one type of knowledge can often lead to an improvement in the other (Rittle-Johnson, Siegler, & Alibali, 2001). Thus, it is still unclear whether it is primarily the conceptual knowledge required to have arithmetic fluency or if it is about executing the specific procedures.

In addition, it has been argued that “algebra” may not be a singular mathematical construct and instead involves a diverse range of conceptual and procedural knowledge (e.g., Kilpatrick & Izsak, 2008; Magruder, 2012). Thus, one limitation of our study is our single measure of algebra fluency, which leaves open the question of how rational number understanding may be related to various aspects of algebra ability. For example, algebra involves manipulating equations and understanding variables (e.g., Kilpatrick & Izsak, 2008), having an understanding of the equal sign (e.g., Knuth, Stephens, McNeil, & Alibali, 2006), having an understanding of the real number system (e.g., Christou & Vosnidou, 2012), and so on. Although our algebra assessment covered a variety of algebraic tasks, it may be that some aspects of rational number knowledge are more relevant for particular aspects of algebra

knowledge than others. In particular, given that different aspects of algebra understanding may differentially rely upon procedural and conceptual competences, it may be that some aspects of algebra are more directly related to the procedural aspects of fraction and decimal understanding (for example, solving step-by-step linear equations; Rittle-Johnson & Star, 2007), while other aspects of algebra may be more directly related to conceptual aspects of fraction and decimal understanding (for example, having a strong understanding of equivalence, including equivalent magnitudes). Thus, this lack of specificity in our measure of algebra fluency may limit the interpretations we can make about what aspect(s) of *algebra* ability may be reliant upon specific aspects of rational number understanding.

Moreover, the current study did not include measures of more general cognitive abilities, like working memory or verbal abilities. Substantial research suggests that working memory may be related to rational number ability in particular (Jordan et al., 2013; Vukovik et al., 2014), leaving open the question of whether these general cognitive abilities may also play a role in the relationship between rational number understanding and algebra ability. Given that we included multiple individual measures in our regression model, which likely correlate with domain-general measures, and still found some of these measures to uniquely predict algebra ability, our findings do point to a strong relationship between rational number understanding and algebra ability. However, future research should include general cognitive measures to rule out the influence of these domain-general aspects of cognitive functioning in contributing to the pattern of results obtained.

Lastly, the current study leaves open the question of whether developmental differences may exist in the relative contribution of arithmetic and magnitude knowledge in both fraction and decimal notation in predicting algebra ability. The relative contributions of different types of

rational number knowledge may depend on the educational stage of the participants as well as the type of notation used. In particular, how arithmetic ability and magnitude knowledge of rational numbers may relate to algebra ability when the child first begins learning algebra, as well as how the relationship may change throughout the learning of algebra is an important open question that may shed light on both the learning of algebra and of rational numbers.

In conclusion, results from the current study suggest that the relationship between rational number knowledge and algebra ability holds even in educated adults and is driven by fraction *and* decimal arithmetic fluency, as well as a more generalized understanding of rational number magnitudes and the symbols used to represent them. The unique roles of fraction and decimal notation highlight that more research is needed to directly compare the use of the two notations for understanding rational number magnitudes and procedures in order to promote both rational number understanding and algebra learning in the classroom.

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**Table 1: Descriptive Statistics**

	Mean (SD)	Correlation Coefficients (r)							
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Algebra CT (sec)	568 (196.5)	-							
(2) Fraction Arithmetic	231 (101.7)	0.575* *	-						
(3) Decimal Arithmetic	417 (137.6)	0.579* *	0.347* *	-					
(4) Fraction RT <sub>adj</sub> (ms)	2243 (1178.8)	0.337* *	0.099	0.29* *	-				
(5) Decimal RT <sub>adj</sub> (ms)	229 (90.1)	0.358* *	0.07	0.274	0.15	-			
(6) Fraction $\beta$ (ms)	-294 (537.2)	-0.11	-0.07	-0.128	-0.440**	-0.06	-		
(7) Decimal $\beta$ (ms)	-44 (68.4)	0.391* *	0.183	0.26	0.22	0.203	-0.164	-	
(8) Fraction SE (ms)	464 (322.3)	0.297* *	0.06	0.27	0.922**	0.192	-0.352* *	0.17	-
(9) Decimal SE (ms)	62 (23.2)	0.389* *	0.12	0.44**	0.34* *	0.62**	0.004	0.255	0.39**

**Table 1:** Descriptive statistics for all measures (N=51). Bivariate Pearson correlations are shown with two-tailed significance indicated as \*\* $p < 0.006$ , \* $p < 0.05$ .

**Table 2: Regression Results**

	<b>Variable</b>	$\beta$	<b>Statistic</b>
Model 1: $R^2=0.29$ , $F(4,46)=4.7$ , $p=0.003$			
<b>Model 1: Magnitude Measures</b>	<i>DvD</i> $\beta$	0.292*	$t(50)=2.25$ , $p=0.029$
	<i>DvD</i> $RT_{adj}$	0.263*	$t(50)=2.06$ , $p=0.045$
	<i>FvF</i> $\beta$	0.076	$t(50)=0.54$ , $p=0.59$
	<i>FvF</i> $RT_{adj}$	0.266	$t(50)=1.88$ , $p=0.066$
Model 2: $R^2=0.495$ , $F(2,48)=23.495$ , $p<0.001$			
<b>Model 2: Arithmetic Measures</b>	<i>Fraction Arithmetic Fluency</i>	0.426*	$t(50)=3.89$ , $p<0.001$
	<i>Decimal Arithmetic Fluency</i>	0.431*	$t(50)=3.94$ , $p<0.001$
Model 3: $R^2=0.590$ , $F(6,44)=10.54$ , $p<0.001$			
<b>Model 3: All Measures</b>	<i>DvD</i> $\beta$	0.172	$t(50)=1.69$ , $p=0.099$
	<i>DvD</i> $RT_{adj}$	0.19	$t(50)=1.89$ , $p=0.065$
	<i>FvF</i> $\beta$	0.079	$t(50)=0.74$ , $p=0.466$
	<i>FvF</i> $RT_{adj}$	0.178	$t(50)=1.60$ , $p=0.117$
	<i>Fraction Arithmetic Fluency</i>	0.416*	$t(50)=4.1$ , $p<0.001$
	<i>Decimal Arithmetic Fluency</i>	0.296*	$t(50)=2.68$ , $p=0.01$

**Table 2:** Results from the regression analyses. There was a significant change from Model 1 to Model 3 ( $\Delta R^2=0.307$ ,  $F(2,42)=16.8$ ,  $p<0.001$ ) and from Model 2 to Model 3 ( $\Delta R^2=0.103$ ,  $F(4,44)=2.81$ ,  $p=0.037$ ).

### Appendix A

Complete list of the 12 Algebra questions

Question:

There are two pipes. The first pipe is  $x$  meters long. The second pipe is  $y$  times as long as the first one. How long is the second pipe?

Question:

In Zedland, total shipping charges to ship an item are given by the equation  $y = 4x + 30$  where  $x$  is the weight in grams and  $y$  is the cost in zeds. If you have 150 zeds, how many grams can you ship?

Question:

Simplify the expression  $2(x + y) - (2x - y)$

Question:

Give two points on the line  $y = x + 2$

Question:

Simplify the expression  $2a^2 \times 3a$

Question:

The table below shows a relation between  $x$  and  $y$

$x$	1	2	3	4	5
$y$	1	3	5	7	9

What is the relation between  $x$  and  $y$ ?

Question:

$3(2x - 1) + 2x = 21$       What is the value of  $x$ ?

Question:

The number of jackets that Haley has is 3 more than the number Anna has. If  $n$  is the number of jackets Haley has, how many jackets does Anna have in terms of  $n$ ?

Question:

$a = 3$  and  $b = -1$       What is the value of  $2a + 3(2 - b)$ ?

Question:

Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?

Question:

Simplify the expression  $4x - x + 7y - 2y$

Question:

If  $\frac{x}{3} > 8$  then what does  $x$  equal?